Two Triangularity Results and Invariants of $(\mathfrak{sp}(p+q,\mathbb{C}), Sp(p)\times Sp(q))$-Modules

Let $\mathcal{O}$ be a special nilpotent co-adjoint orbit and write $A(\mathcal{O})$ for the group of components of the centralizer of an element in $\mathcal{O}$. The Springer correspondence attaches to $(\mathcal{O}, e \in \hat{A}(\mathcal{O}))$ an irreducible W-modules $Sp(\mathcal{O})$. $Sp(\mathcal{O})$ admits two basis consisting of W-harmonic polynomials. One basis consists of Goldie rank polynomials $\{P_I : I \text{ is a primitive ideal with inf. char.} \rho \text{ and } AV(I) = \bar{\mathcal{O}}\}$. The second basis consists of polynomials parametrized by orbital varieties (irreducible components of $\bar{\mathcal{O}} \cap \mathfrak{n}$). We write $\{P_\Upsilon : \Upsilon \text{ is orbital for } \mathcal{O}\}$. McGovern defined a combinatorial order on orbital varieties so that, in that order, the matrix that relates the basis of Goldie rank polynomials to $\{P_\Upsilon\}$ is upper triangular. On the other hand, Trapa, using the geometry of characteristic cycles of Harish-Chandra modules of real forms, defined different orders on the set $\{\Upsilon\}$ and concluded that the relation between basis of $Sp(\mathcal{O})$, in those orders, is upper triangular. In this talk, in the context of $\mathfrak{g}_\mathbb{R} = \mathfrak{sp}(p,q)$, we investigate the relation between these orders and derive consequences relevant to the computation of invariants of Harish-Chandra modules. In particular, we address a question posted by Trapa on the shape of the Leading term cycle of such modules.