Computer Computations in Representation Theory I: Finite Groups and SL(2)

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1 Finite Groups

Let G be a finite group, and $GL(n, \mathbb{C})$ the group of $n \times n$ invertible (determinant $\neq 0$) matrices over $\mathbb{C}$.

Definition 1.1 *A representation of G is a group homomorphism $\pi : G \to GL(n, \mathbb{C})$.*

That is: π is an action of G on $V = \mathbb{C}^n$. We sometimes write (π, V).

Example: $G = S_n$ (the symmetric group), $V = \mathbb{C}^n$. For $\sigma \in G$ let

$$\rho(\sigma)(z_1, \ldots, z_n) = (z_{\sigma(1)}, \ldots, z_{\sigma(n)})$$

This is the *reflection* representation; $\rho(G)$ is the permutation matrices.

For example if $n = 3$,

$$\rho(1\,2) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \rho(1\,2\,3) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Note that the line $V_1 = \mathbb{C} < (1, 1, \ldots, 1) >$ is invariant under this action; the orthogonal complement $V_2 = \{\vec{z} \mid \sum z_i = 0\}$ is also taken to itself by this action. We say $(\pi, V) = (\pi_1, V_1) \oplus (\pi_2, V_2)$.

Definition 1.2 *A representation (π, V) is reducible if there is a subspace*

$$0 \subsetneq W \subsetneq V$$

such that

$$\pi(g)W \subset W \text{ for all } g \in G$$

Otherwise we say (π, V) is irreducible.

Equivalently $V = V_1 \oplus V_2$, $\pi = \pi_1 \oplus \pi_2$.

In the example above (π_1, V_1) and (π_2, V_2) are irreducible representations of S_n.

Definition 1.3 *Two representations are isomorphic, or equivalent, if they are the same up to a change of basis. That is* (π, V) *is isomorphic to* (π', V) *if there exists* $X \in GL(n, \mathbb{C})$ *such that* $\pi'(g) = X\pi(g)X^{-1}$ *for all* $g \in G$.

Theorem 1.4 (Frobenius) *The number of equivalence classes of irreducible representations of G is the same as the number of conjugacy classes of G.*

Write $\pi_1, \ldots, \pi_n$ for the irreducible representations. Then $\sum_i dim(\pi_i)^2 = |G|$.

A representation π is determined by its character, the conjugation invariant function $\theta_\pi(g) = Tr(\pi(g))$.

See the beautiful book by Serre, Linear Representations of Finite Groups [5], pp. 1–19.

The characters may be computed explicitly: this is the *character table*. See The Atlas of Finite Groups [3].

Basic Problem: Find models of representations of G. That is given a representation π, for example by its character, and generators $\{g_i\}$, find the matrices $\{\pi(g_i)\}$ explicitly.

Even by computer this can be a difficult problem (see below).

Note if G has generators $g_1, \ldots, g_n$ and relations, then a representation is given by n matrices A_i satisfying the same relations (take $\pi(g_i) = A_i$).

Example: a 64-dimensional representation of $W(E_6)$

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pi(g_1):
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[0 0 0 -85/144 85/144 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 137/432 115/432 0 0 -11/36 5/36 0 0 -31/36 1/
[0 0 0 17/48 -17/48 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 23/144 61/144 0 0 1/12 -1/4 0 0 1/60 -17/20 0 0
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[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -5/6 -2/3 0 0 0 0 0 0 0 0 0 0 0 -1/2
[0 0 0 1/2 -1/2 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1/6 -1/6 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 1/12 -1/1
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[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -5/6 -2/3 0 0 0 0 0 0 0 0 0 0
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[0 0 0 11/48 -11/48 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 179/144 121/144 0 0 7/3 11/6 0 0 7/15 11/30 0 0
[0 0 0 -275/16 275/16 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -275/48 1325/16 0 0 0 925/6 0 0 0 185/6 0 0 -
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Example: The Monster

The finite simple groups are classified into infinite families (more on this in Lecture II) and 26 *sporadic* groups. The largest sporadic group is the Monster M, with order

$$808,017,424,794,512,875,886,459,904,961,\\ 710,757,005,754,368,000,000,000 \simeq 10^{54}$$

It has 194 irreducible representations. (This is an astonishingly small number of representations for a group this size. The alternating group A_{18} has order

$$3,201,186,852,864,000 \simeq |M|/10^{38}$$

and 200 representations.) The smallest non–trivial representation has dimension $196,883$.

$$j(q) = \frac{1}{q} + 196,884q + 21,493,760q^2 + \dots$$

Here $j(z)$ is the Weierstrass j–function and $q = e^{2\pi i z}$.

Monstrous Moonshine [4] is the observation that

$$196,884 = 1 + 196,883$$

i.e. the coefficient of q is $j(q)$ is the sum of the dimensions of the two smallest irreducible representations of M (also higher terms).

Richard Borcherds [1] proved that all of the coefficients of $j(q)$ are computed in terms of the dimensions of representation of the Monster group (part of the work for which he won the Fields Medal in 1998).

Dimensions of representations of the monster:

π_1	1
π_2	$196,883$
π_3	$21,296,876$
π_4	$842,609,326$
π_5	$18,538,750,076$
π_6	$19,360,062,527$
π_7	$293,553,734,298$
π_8	$3,879,214,937,598$
π_9	$36,173,193,327,999$
	...
π_{194}	$258,823,477,531,055,064,045,234,375$

2 $SL(2, \mathbb{R})$

Let $SL(2, \mathbb{R})$ be the group of 2×2 invertible matrices of determinant one with real coefficients. Note that G is a topological space (in fact a manifold).

A finite dimensional representation of G is a continuous group homomorphism $\pi : G \to GL(n, \mathbb{C})$.

Example: $\pi(g) = g$ (the tautological representation)

Example:

$$\pi \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ad + bc & -ac & bd \\ -2ab & a & -b \\ 2cd & -c & d \end{pmatrix}$$

Now G has *interesting infinite dimensional representations*:

Example: $V = L^2(\mathbb{R}^2)$, $\pi(g)f(v) = f(g^{-1}v)$.

Note that $(\pi(g)f_1, \pi(g)f_2) = (f_1, f_2)$ for all $f_1, f_2 \in V, g \in G$.

Example: $V = L^2(\mathbb{R})$. Fix $\nu \in \mathbb{C}$. Then

$$\pi_\nu(g)(f)(x) = |-bx+d|^{-1-\nu} f(g \cdot x)$$

Here $x \to g \cdot x$ is the action of $SL(2,\mathbb{R})$ on $\mathbb{R}$ by linear fractional transformations:

$$g \cdot x = \frac{ax-c}{-bx+d}$$

if $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

These representations are *unitary* if $\nu \in i\mathbb{R}$:

$$(\pi(g)f_1, \pi(g)f_2) = (f_1, f_2)$$

with the usual inner product, i.e.

$$(f_1, f_2) = \int_{\mathbb{R}} f_1(x)\overline{f_2}(x)\, dx$$

Suppose V is a Hilbert space.

Definition: A representation π of G on V is a homomorphism π of G into the bounded linear operators (with bounded inverses) on V, such that the map $g \to \pi(g)v \in V$ is a continuous map from G to V.

The representation π is unitary if

$$(\pi(g)v, \pi(g)w) = (v, w)$$

for all $v, w \in V, g \in G$, where $(,)$ is some Hilbert space structure on V.

π is irreducible if it contains no *closed* invariant subspace.

In our example:

π_ν irreducible unless $\nu \in 2\mathbb{Z} + 1$.

π_ν is unitary for the given Hilbert space structure if and only if $\nu \in i\mathbb{R}$. It is also unitary if $-1 \leq \nu \leq 1$ (for some other inner product).

Definition 2.1 *The unitary dual $\widehat{G}$ of G is the set of (equivalence classes of) irreducible unitary representations of G.*

Typically $\hat{G}$ is uncountable. For example classical Fourier analysis amounts to the fact that $\widehat{\mathbb{R}} \simeq \mathbb{R}$; $y \in \mathbb{R}$ corresponds to the unitary representation $\chi_y : x \to e^{ixy}$ of $\mathbb{R}$.

Note that $L^2(\mathbb{R})$ is a representation of $\mathbb{R}$, by $\pi(x)(f)(y) = f(x+y)$. Fourier inversion

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(y) e^{ixy}\, dy$$

says that $L^2(\mathbb{R})$ is isomorphic, as a representation of $\mathbb{R}$, to the "continuous direct sum" of the irreducible one-dimensional representations χ_y.

3 Other Groups

$SL(2,\mathbb{R})$ is an example of a *Lie* group, named after the Norwegian mathematician Sophus Lie (1842-1899). Lie introduced them to study symmetry in differential equations and other settings. It is a topological group.

Other examples:

The **general linear group** $GL(n,\mathbb{R})$: all $n \times n$ invertible real matrices. The special linear group $SL(n,\mathbb{R})$ is the subgroup of matrices of determinant 1.

The **symplectic group** $Sp(2n, \mathbb{R})$: the $2n \times 2n$ real matrices satisfying

$$gJg^t = J$$

where $J = \begin{pmatrix} 0 & I_n \\ -I_n & 0 \end{pmatrix}$.

The **orthogonal group** $SO(m, \mathbb{R})$: the $m \times m$ real matrices satisfying $det(g) = 1$ and

$$gKg^t = K$$

where

$$K = \begin{cases} \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix} & m = 2n \\ \begin{pmatrix} 0 & I_n & \\ I_n & 0 & \\ & & 1 \end{pmatrix} & n = 2n + 1 \end{cases}$$

(This is the *split* orthogonal group, not the compact one.)

3.1 Finite Groups of Lie Type

The groups $SL(n), Sp(2n)$ and $SO(n)$ can be defined over any field. Over a finite field they are *finite groups of Lie Type.* Except for finitely many values of n and the order q of the finite field these groups, modulo their finite center, are simple.

The finite simple groups are: $\mathbb{Z}/p\mathbb{Z}, A_n$ $(n \geq 5)$, the finite simple groups of Lie type, and 26 sporadic simple groups.

A lot of information about these groups and their representations may be found in the Atlas of Finite Groups [3].

The representation theory of finite groups of Lie type is a beautiful subject, developed primarily by George Lusztig [2].

4 Atlas of Lie Groups

Proposal: Create an on-line ATLAS OF LIE GROUPS

The idea is modelled on the Atlas of Finite Groups. It will contain information about all simple real Lie groups (and possibly groups over other fields?) It will allow the user to look up information on the representation theory of a given group. I will discuss other more exotic Lie groups in Lecture II.

The first goal is to give a tool to look up the unitary spherical representations of any split real group. This will be discussed further in Lecture III.

5 Computing the Unitary Dual by Computer

Theorem 5.1 (David Vogan) *Let G be a real Lie group, such as $SL(n,\mathbb{R}), Sp(2n,\mathbb{R})$ or $SO(n,\mathbb{R})$. Then there is a finite algorithm to compute the unitary dual of G.*

There is a very big difference between a theorem asserting the existence of an algorithm, and a computer program.

Proposal: Write a computer program to compute the unitary dual of various Lie groups.

I will discuss a very special case of such a program in Lecture III. k

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